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AUTHOR(S): B. L. Kortegaard

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Los Alamos

Los Alamos National Laboratory
Los Alamos, New Mexico 87545

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# Superfine laser position control using statistically enhanced resolution in real time

B. L. Kortegaard

University of California, Los Alamos National Laboratory
P. O. Box 1663, Mail Stop E529, Los Alamos, New Mexico 87545

## Abstract

An electronic control system is described which analyzes digitized TV images to simultaneously position 96 time-and-space multiplexed beams for a large KrF laser system. Degradation of position resolution due to the intervals between digitization is discussed, and improvement of this resolution by using inherent system noise is demonstrated. The methods shown resolve arbritary intensity boundaries to a small fraction of the discrete sample spacing.

## Introduction

Aurora, the 0.2484 micron kilojoule-class KrF laser system being developed at Los Alamos as part of the national alternative energy program, has a 5 ns oscillator pulse length but a main amplifier pump time of 500 ns. The oscillator pulse is therefore cross-sectioned into a 96 beam array, with the individual beams then sent over different length paths to a 12 by 8 mirror array. The angle of incidence of each individual mirror with respect to its beam is set so that the 96 beam elements are all directed through the optical system to the main amplifier. Each beam fills the 44 inch square main amplifier but arrives at times spaced 5 ns apart, effectively as a single 480 ns pulse.

To control these mirrors, the beams they direct are split and imaged on a TV camera, still as a 12 by 8 array. The X-Y position of each element of the image has a one-to-one correspondence with the angle of the associated beam at the amplifier input aperture. The system is controlled by analyzing the image, and directing stepper motors to move the mirrors until the elements of the image are at the desired X-Y coordinates. (Figure 1)

Each element of the image falls inside a 48x64 pixel (picture element) window and the resolution of its position within that window determines the resolution of the control system. The pixels are distinguished from one another by the (x,y) coordinates of the image location they represent, and the contents of each pixel is the digitized value of the intensity of that location when last sampled. The x coordinates are separated by discrete intervals, corresponding to the time interval between the 520 intensity samples of each horizontal scan, and the y coordinates are separated by intervals corresponding to the vertical distance between the 480 interlaced horizontal lines. The actual value of these intervals is determined by the dimensions of the area imaged on the camera, but are normalized to integer values separated by unity increments ($\Delta x, \Delta y$), for analysis. When we speak of resolution to a fraction of a pixel, we mean resolution to a fraction of $\Delta x$ or $\Delta y$.

The final phase of the KrF Program will require a position resolution equivalent to $\pm 0.01$ pixels, which corresponds to an angle of $\pm 2$ microradians. Noise-enhanced resolution will be one of the primary tools for minimizing alignment time. The present system uses this tool and has a resolution of $\pm 0.05$ pixels ($\pm 10$ microradians). This quantitatively demonstrates the effectiveness of statistically enhanced resolution, while adequately controlling the beam positions for the Main Amplifier phase of the program and providing data for the design of the final system.

Resolution for this type of system is determined by the number of evenly spaced digitized samples, the accuracy of the statistical model for the beam-image and random noise fields, and the validity of the beam-position estimation criterion. The analysis and discussion of these factors and how they may be used to enhance the expected resolution of a discrete sampler comprise the remainder of this paper.

## Effect of spatial image-sampling on beam resolvability

The vidicon image is a two-dimensional distribution of light intensities $B_i(x,y,t)$ which represent the cross-sectional energy distribution of the various laser beams $L_i$. The problem of angular control of the beams involves finding the imaged beam positions as indicated by these intensities. The factors affecting the position resolution of any one beam are the same as for any other, and this analysis is therefore restricted to minimizing the effect of uncertainties in determining the position of a single beam, in its 48x64 pixel window.

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$B(x,y,t)$ produces a bounded image. That is, there exists some positive integers $B_y$ and $B_y$ such that

$$B(x,y,t) = 0 \text{ if } |x-X_o|>B_x \text{ or } |y-Y_o|>B_y, \tag{1}$$

where $(X_o,Y_o)$ are the arithmetic mean of coordinates in the sample space that are within the imaged beam energy distribution.

The beam magnitude and density are random processes, although the magnitude is non-stationary. However, over any fixed interval T during which the image is analyzed:

$$\int_n^{n+T} \frac{B(x-X_i,y-Y_i,t)dt}{B_n \text{ Max}} = \int_m^{m+T} \frac{B(x-X_i,y-Y_i,t)dt}{B_m \text{ Max}}, \text{ for all } n,m \tag{2a}$$

where Bn and Bm are the average maximum values for the function, which are here defined with respect to the fixed coordinates $(X_i, Y_i)$. We will call these averages $I(x,y)$.

$$I(x,y) = \int_o^T \frac{B(x,y,t)dt}{B_o \text{ Max}} \tag{2b}$$

(The accuracy of this assumption, and the optimum interval T, are currently under investigation, using this control system. They are at least accurate enough for the overall resolution demonstrated.)

As the mirrors pointing the beam are moved, its imaged energy distribution moves about the sample space and its shape may change.
In general:

$$I(x-X_1,y-Y_1) \neq I(x-X_2,y-Y_2), \text{ if } (X_1,Y_1) \neq (X_2,Y_2), \tag{3}$$

although this normalized, time-averaged, cross-sectional intensity distribution is invariant for a given position, and changes with position are continuous and slowly varying. That is, for beam movements of a pixel or so, the change in shape is insignificant to its analysis.

The image capture system (Figure 1a) uses standard printed circuit boards from Imaging Technology in an 8086 microprocessor-controlled multibus environment. It is configured to select either single video frames, or an average of up to 256 separate frames, for analysis. By analyzing and storing the area of interest of one image while the next is being captured, we can actually average our results over as many frames as we wish.

Nyquist sampling cannot be usefully applied to this sample space because the spacing is too far apart for the resolution precision we require. The measuring intervals are closer in y than x for our system, but in either case the Fourier transform has significant components with shorter periods, than that of the highest valid Fourier component present in the sample.

## Position determination from the arithmetic mean

Since approximation methods will therefore be required, we have chosen a method which is fast and cost effective, yet which still has adequate precision in reproducibly determining beam position. We decide which pixels are included within the bounded image, find the arithmetic mean of their coordinates and call that mean a measure of the beam position. Since $I(x-X_j,y-Y_k)$ is stationary, and changes with $(X_j,Y_k)$ are continuous and slowly varying, this measure is a single-valued function of beam position, and is repeatable to the degree the mean is resolved precisely. Treating the mean as a random variable, this precision is determined by its density and distribution functions which are themselves determined by the measurement method and the boundary estimation criterion.

## $(X_m, Y_m)$ as a function of beam position

A pixel $(X_i, Y_j)$ is a Beam Pixel when

$$I(X_i, Y_j) \geq I_b \tag{4a}$$

where

$$I_b = \text{the intensity threshold at which the beam is considered to exist.} \tag{4b}$$

By making $I_b$ a function of beam energy, the method is extended to finding Band Pixels, those pixels which are within equi-intensity lines representing the bounds of a given energy band. The problem of reproducing the entire beam could be reduced to establishing its appropriate contour line, but we are concerned here only with finding the arithmetic mean of the coordinates within the beam threshold.

For our laser beams, where the imaged equi-intensity lines are everywhere convex, coordinates for the measured beam position $(X_m, Y_m)$ are:

$$X_m = \frac{\sum_{x=o}^{X_i - X_o} R_x(X_o + x)}{N} = \frac{\sum_{y=o}^{Y_i - Y_o} C_y(X_y + (C_y - 1)/2)}{N} \tag{5a}$$

$$Y_m = \frac{\sum_{y=o}^{Y_i - Y_o} C_y(Y_o + y)}{N} = \frac{\sum_{x=o}^{X_i - X_o} R_x(Y_x + (R_x - 1)/2)}{N} \tag{5b}$$

where

$X_o, X_i$ = First and last column with Beam Pixels
$Y_o, Y_i$ = First and last row with Beam Pixels
$C_y$ = Number of columns in yth row with Beam Pixels
$R_x$ = Number of rows in xth column with Beam Pixels
$X_y$ = Lowest X coordinate of Beam Pixel in yth row
$Y_x$ = Lowest Y coordinate of Beam Pixel in xth column
$N$ = Total number of Beam Pixels

$X_m$ and $Y_m$ are not continuous over $I(x,y)$ since the pixels are physically separated by $\Delta x$, $\Delta y$, and the only place the derivative is non-zero it is discontinuous. The variation of $X_m$ with movement of $I(x,y)$ across the image plane is actually a distribution with discrete changes.

As shown in Figure 2, the density function is a series of delta functions, and the distribution function is an uneven staircase. If the beam were to move only a slight distance, and in the x direction, the weight of the delta function could be expressed as

$$\Delta X_m \sim \frac{\sum_{y=o}^{Y_i - Y_o} \left[\left(X_y - X_m + \frac{2C_y - 1}{2}\right) \Delta C_y + C_y \Delta X_y\right]}{N} \tag{6}$$

A beam with a large number of pixels, spread over many rows and columns might then have a distribution which is almost continuous. A beam having only a few beam pixels will have a resolution worse than ±.3 pixels or so, even in a noise free environment. A beam smaller than $\Delta x$ or $\Delta y$ might easily be disastrous since its resolution error can equal the coordinates of the nearest pixel.

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In our system, the beam is not only small, which is not really necessary, but it may also be rectangular. With a rectangular beam the resolution will be worse than ±0.5 pixels, since the side may be normal to the direction of movement. As we require much greater precision than this, we need some type of resolution enhancement.

When $I_b$ is such that adjacent pixel intensities fall on either side of it, the displacement can be approximated through linear interpolation. Let $I_q$ denote $I_{n-1}$ if the intensity decreases to the right and $I_{n+1}$ if the intensity increases to the right. Then the approximation is given by:

$$P_n = \begin{cases} \dfrac{(I_q - I_b)}{(I_q - I_n)} & ; \quad I_q \ge I_b > I_n \\ 1 & ; \quad I_n > I_b \\ 0 & ; \quad I_b \ge I_q \end{cases} \tag{7a}$$

The mean can be modified accordingly.

$$(X_m, Y_m) = \left( \frac{\sum_{j=0}^{R-1} \sum_{i=0}^{C_j-1} (X_j + i)\, P_{ji}}{\sum_{j=0}^{R-1} \sum_{i=0}^{C_j-1} P_{ji}} , \frac{\sum_{j=0}^{C-1} \sum_{i=0}^{R_j-1} (Y_j + i) P_{ji}}{\sum_{j=0}^{R-1} \sum_{i=0}^{C_j-1} P_{ji}} \right) \tag{7b}$$

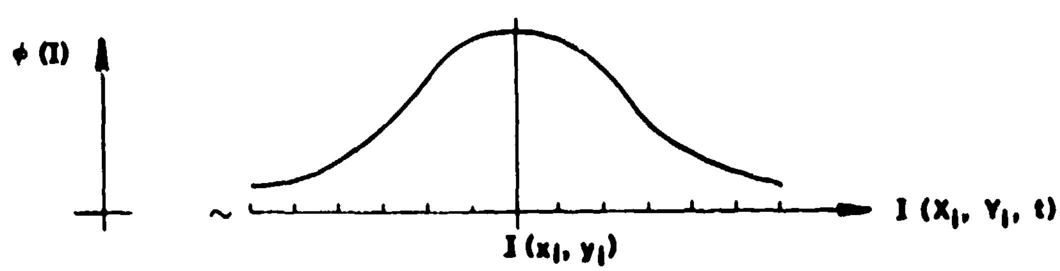
In this way the density function is made continuous, and the function is accurate to the degree the beam variation can be approximated as a straight line. For some $I_b$ selections, this assumption is probably true to ±0.2 pixels for all beam shapes, and using a 4-bit successive approximation algorithm the worst-case resolution is potentially ±0.1 pixels with reasonable program speed. This is still an order of magnitude less precision than we will eventually require even for only position resolution, however, and the straight-line assumption is not valid for many parts of the beam.

What we really want is a method fast enough so that we may control the positioning system in real-time, physically realizable for our immediate objectives and with the potential to find $(X_m, Y_m)$ to whatever precision we may wish.

## The effect of random noise on the distribution function of $(X_m, Y_m)$

The method we choose consists of using a system with random noise to do a series of Bernoulli trials for each pixel, giving the coordinate a weight of 1 if its measured intensity exceeds the boundary specified. After a sufficiently large number of samples, the weight for each coordinate will approach the expected number of times the sample intensity will have exceeded the threshold. This expected number is proportional to the weight the coordinate should be given in the arithmetic mean.

Because of noise, the ADC sampled measurements of $I(X,Y,t)$ for a fixed beam location form a discrete sample space. If $\phi(I)$ represents a normalized many-sample frequency count of the intensities measured at $X_i, Y_i$ for a fixed beam position, the plot is approximately:



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Assume the frequency distribution for these counts form a normal density function,

$$\phi(I) = \frac{\exp(-(I-I_o)^2/2\sigma^2)}{\sqrt{2\Pi}} \qquad (8a)$$

and that its integral is a normal distribution function.

$$\Phi(I') = \int_{-\infty}^{I'} \phi(I)\,dI \longrightarrow (1 \text{ as } I \longrightarrow \infty) \qquad (8b)$$

Define a binary function on this space such that

$$\begin{aligned} W(X_i,Y_i)_j &= 1 \quad ; \quad I\,(X_i,Y_i,T_j) \geq I_b \\ &= 0 \quad ; \quad I\,(X_i,Y_i,T_j) < I_b \end{aligned} \qquad (9a)$$

For a fixed beam position, let $p_j$ be defined by

$$p_j = P\,\{W(X_i,Y_i)_j = 1\} = \int_{I_b}^{\infty} \phi(I(X_i,Y_i)\,dI = 1 - \Phi\,(I_b) \qquad (9b)$$

since the measurements are independent.

For enough samples of the intensity at that position,

$$\sum_{j=1}^{n} W(X_i,Y_i)_j \longrightarrow n\,p_i \qquad (10)$$

in a manner corresponding to the DeMoivre-Laplace Limit Theorem

$$P\left\{ \Delta_1 \leq \frac{\sum_{j=1}^{n} W(X_i,Y_i)_j - n\,p_i}{\sqrt{n\,p_i(1-p_i)}} \leq \Delta_2 \right\} \longrightarrow \Phi\,(\Delta_2) - \Phi\,(\Delta_1) \qquad (11)$$

Our method of finding the arithmetic mean is then

$$(X_m,Y_m) = \frac{\sum_{i=1}^{W} (X_i,Y_i)p_i}{\sum_{i=1}^{W} p_i} \qquad (12)$$

where W is the total number of pixels in the window. $(X_m,Y_m)$ is then a continuous, single valued function of beam position.

The above reasoning is illustrated in Figure 3. If the expected intensity difference between adjacent pixels was 2 times the standard deviation, the linearity of the distribution function would still be clearly increasing at all times, and the averaged values would converge to within a ±0.04 pixel precision with a probability of 0.9, in 1024 trials. The following example gives the details of this calculation.

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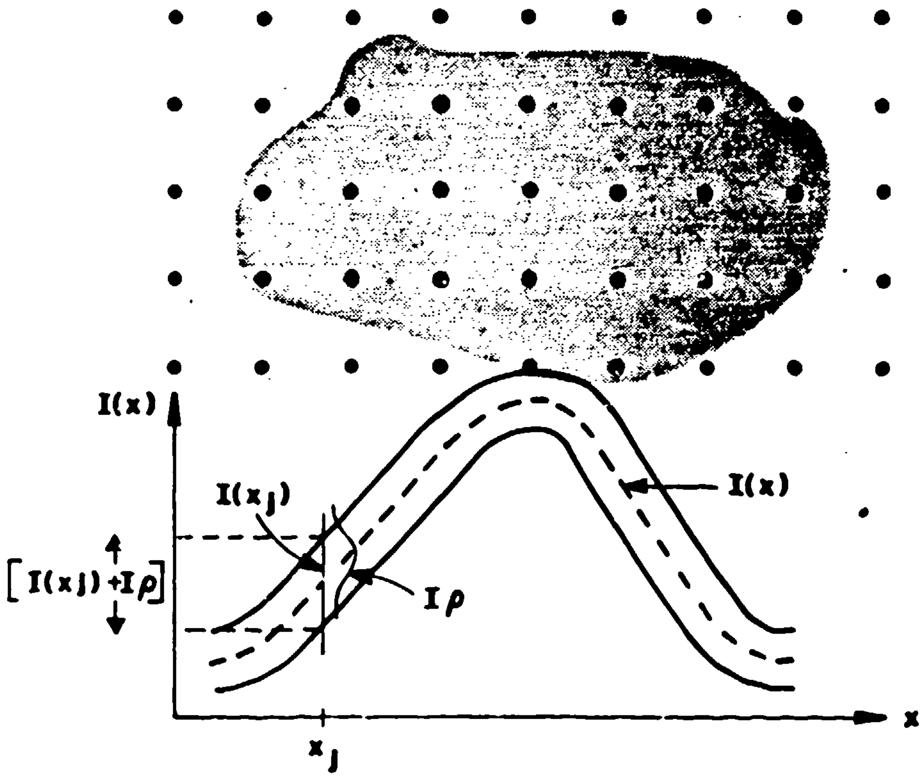
**Example:**




For fixed $X_j$, define a weight function

$$W(X_j) = 0 \quad ; \quad I(X_j) + I_p < I_b = I \text{ threshold} \tag{13a}$$

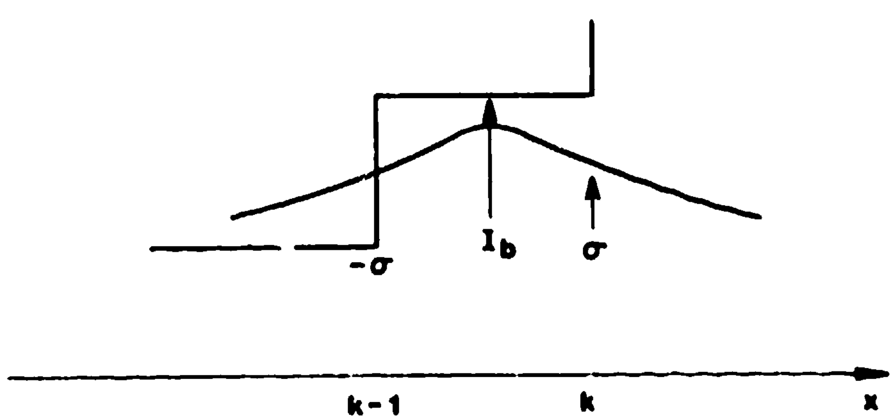
$$W(X_j) = 1 \quad ; \quad I(X_j) + I_p \geq I_b \tag{13b}$$

where $I_p + I(X_j)$ is the value measured, $I(X_j)$ is the true intensity and $I_p$ the noise component of the value measured.

If $I(X_j) > I_p$, then the probabilities that the weight function is 1 or 0 are given by

$$P\{W(X_j) = 1\} = \int_{-\infty}^{I(X_j)-I_b} \phi(I_p)dI_p = \Phi\left(I(X_j - I_b)\right) \tag{14a}$$

$$P\{W(X_j) = 0\} = 1 - \Phi\left(I(X_j) - I_b\right) \tag{14b}$$



Let $X_{k-1}$ and $X_k$ be adjacent points in our sample space whose expected measured intensities are such that

$$I(X_k) - I(X_{k-1}) = 2\sigma;\ I(X_k) = I_b + \sigma;\ I(X_{k-1}) = I_b - \sigma \tag{15}$$

where $\sigma$ is the standard deviation of the random noise distribution $I_p$, then define $p_k$ by

$$p_k = E(W(X_k)) = \Phi(\sigma) = .8413 \tag{16}$$

If

$$I(X_k) \rightarrow I_j + \sigma(1 \pm .02) \tag{17}$$

which corresponds to ± .01 pixel spacing

then

$$E(W(X_k)) \rightarrow .8461 \text{ as } X_k \rightarrow 1.02\sigma \tag{18a}$$

$$E(W(X_k)) \rightarrow .8365 \text{ as } X_k \rightarrow .98\sigma \tag{18b}$$

Then using the DeMoivre Laplace theorem with

$$\Delta_1 = (n\,E(W(X_k + .98\sigma)) - n\,p_k)/\sqrt{n\,p_k(1-p_k)} = -.0131\sqrt{n} \tag{19a}$$

$$\Delta_2 = (n\,E(W(X_k + 1.02\sigma)) - n\,p_k)/\sqrt{n\,p_k(1-p_k)} = .0131\sqrt{n} \tag{19b}$$

The probability that the measured weights of $X_k$ are correct to ± .01 pixel spacing is given by

$$P\left\{\Delta_1 \le \frac{\sum_{j=1}^{n} W(X_i, Y_i)_j - n\,p_i}{\sqrt{n\,p_i(1-p_i)}} \le \Delta_2\right\} \tag{20a}$$

$$= \Phi(.0131\sqrt{n}) - \Phi(-.0131\sqrt{n}) \tag{20b}$$

In this same manner the following probabilities were calculated for different numbers of sample frames and different pixel resolutions.

| Number of Frames n | P {± .01} | P {± .02} | P {± .04} |
|---|---|---|---|
| 1 | .01 | .02 | .04 |
| 4 | .02 | .04 | .08 |
| 16 | .04 | .09 | .16 |
| 64 | .08 | .17 | .32 |
| 256 | .16 | .33 | .59 |
| 1024 | .33 | .61 | .89 |

To close with a visual example, Figure (4a) shows an image with 14 gray levels of signal and noise. Figure (4b) shows the image reduction to 5 gray levels of signal alone, through integration. Figure (4c) shows the 51 gray levels extended to 90 gray levels, through the above method of statistical interpolation. Figures (4d) - (4f) extend the method to show the recovery and analysis of an image apparently completely lost in noise.

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# AURORA REQUIRES RAPID ALIGNMENT OF MANY BEAMS

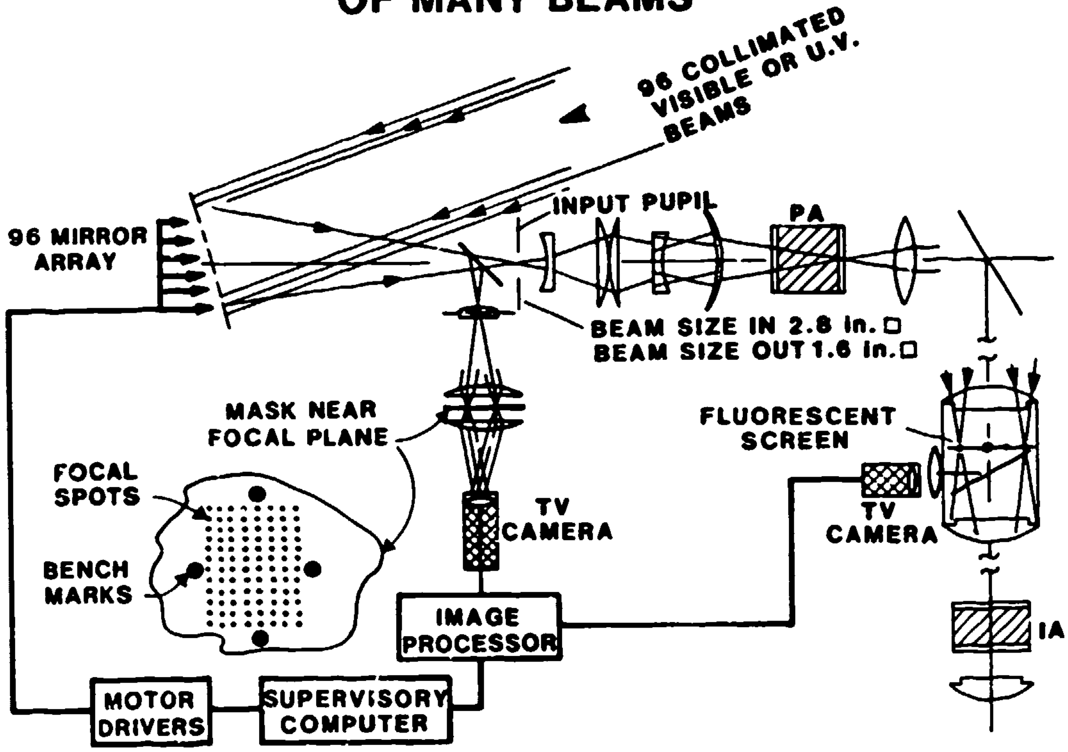


FIGURE 1

# STANDARD COMMERCIAL MODULES ARE COMBINED WITH OUR CUSTOM MOTOR DRIVER PC DESIGN TO MAKE THE COMPUTER/IMAGE ANALYZER

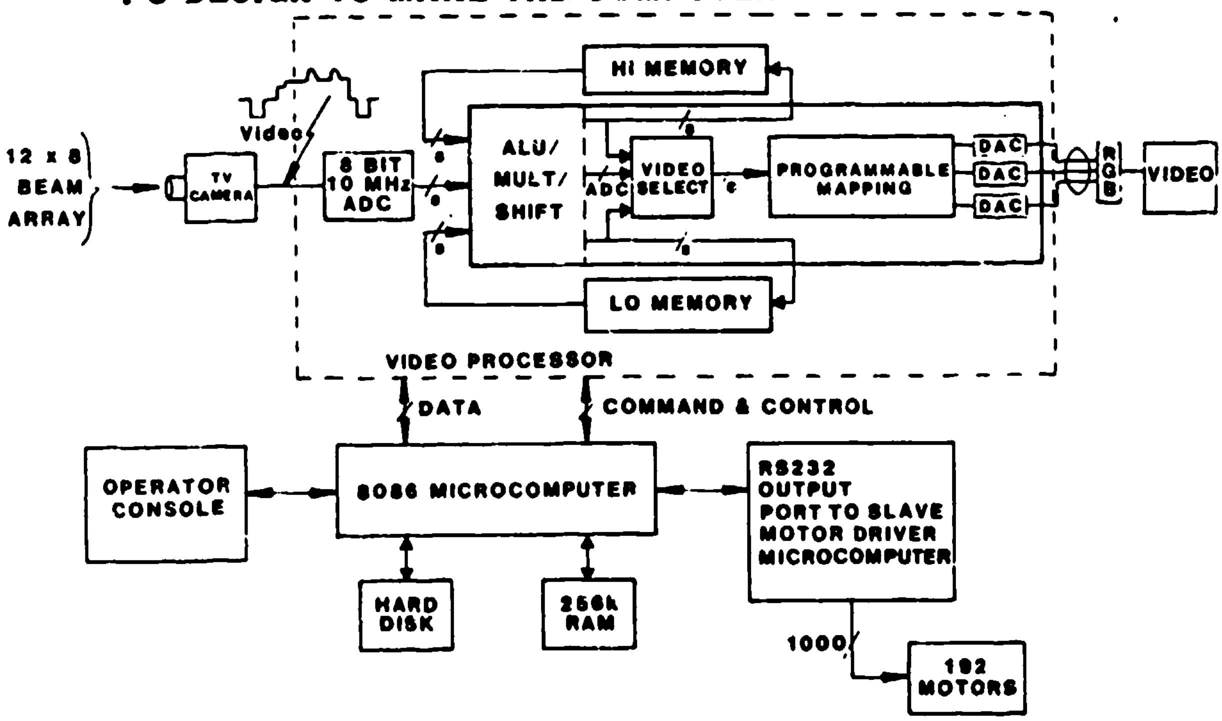


FIGURE 1A

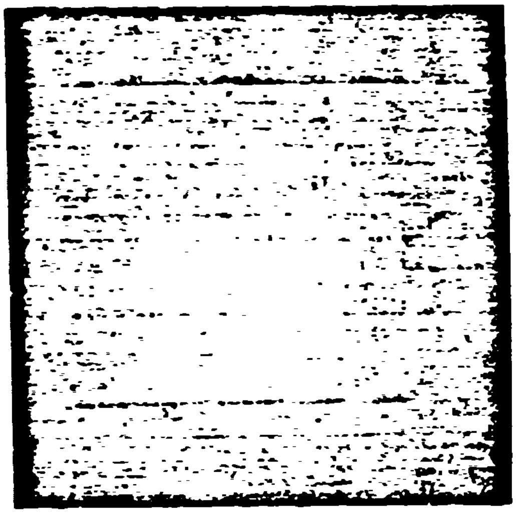

Figure 4a: A video image containing 14 gray levels of signal and noise.

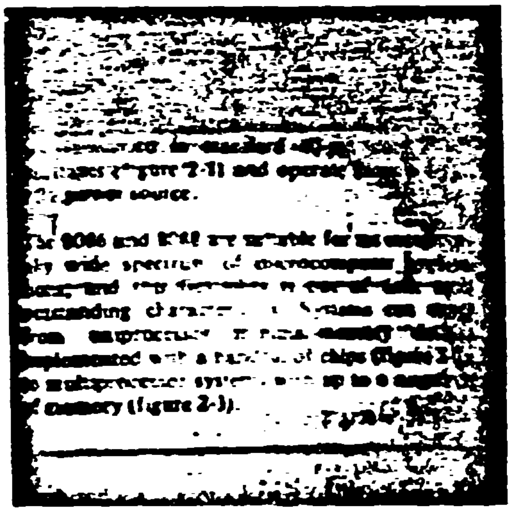

Figure 4b: Noise removal through integration results in 5 gray levels of pure signal.

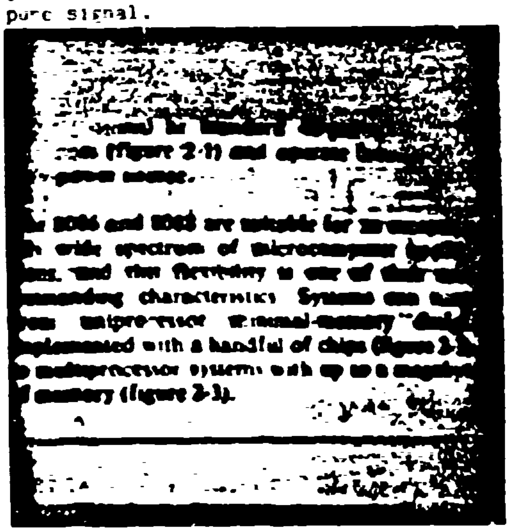

Figure 4c: Statistical interpolation extends the 5 gray levels to 90 gray levels.

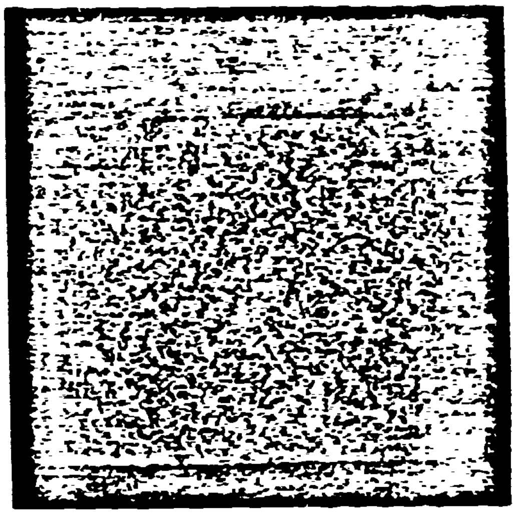

Figure 4d: Low level image buried in noise.

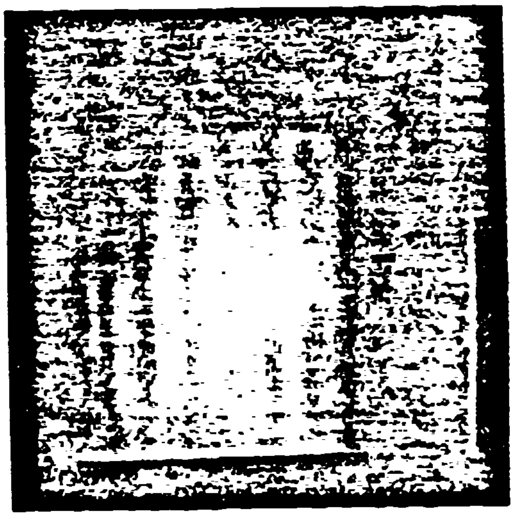

Figure 4e: Image after statistical interpolation and integration.

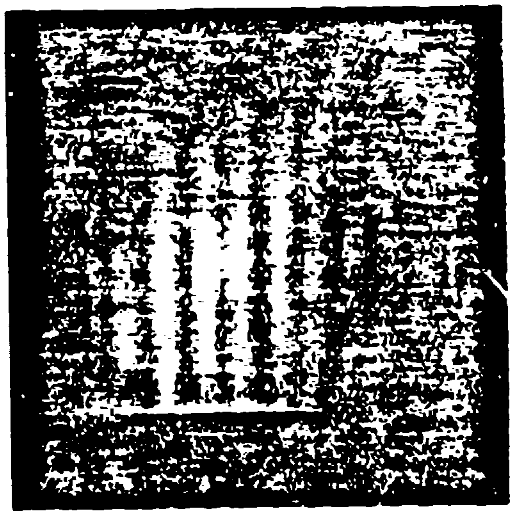

Figure 4f: Enhancement of selected energy band of interpolated image.

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## Summary

The problem of precisely positioning a large array of laser beams with a cost/performance optimal measuring and control system requires making maximum use of the available video information bandwidth. The closed loop response time of the control system is consistent with multiple-frame image sampling at video frequencies, so the advantages gained from statistical inference can be used to gain the necessary precision of resolution with algorithms that use the field acquisition time to accomplish analysis and control.

The accelerating availability of useful computer aided engineering (CAE) support, and the related growth in field programmable devices such as PALs, make the extension of these techniques to higher frame rates and more sophisticated analysis very promising.

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